

Introduction to continuous-time movement modeling

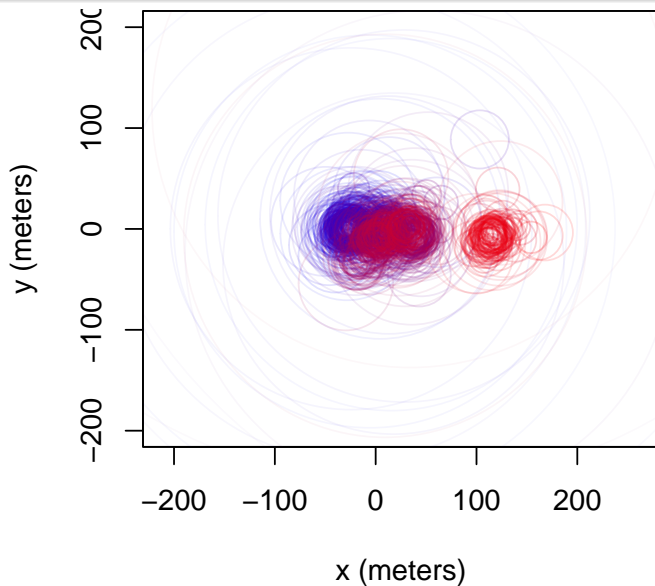
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University of Central Florida



CASUS

2024-06-19



Animal tracking data analysis goals in this workshop

Animal tracking data analysis goals in this workshop

- Account for autocorrelation

Animal tracking data analysis goals in this workshop

- Account for autocorrelation (stochastic process models)

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- Account for sampling irregularity & mismatch

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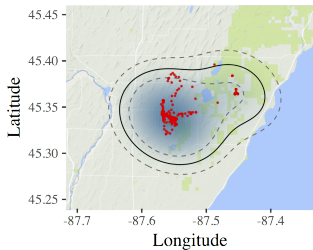
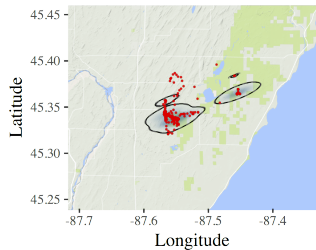
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- Reducing bias and error as much as possible (MVU, BLUE, debiased estimators)

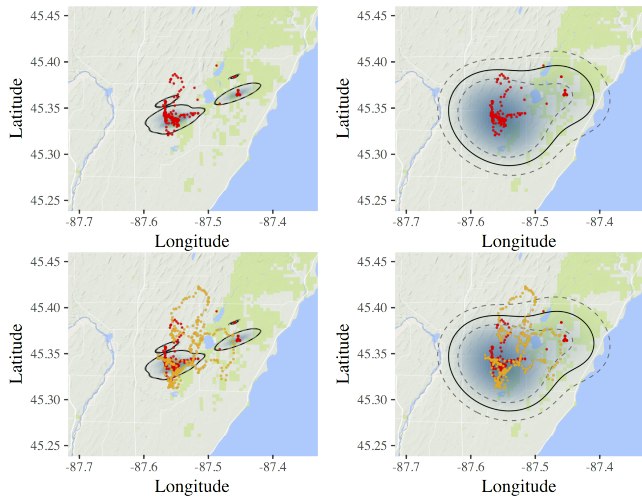
Motivating example: Neglecting autocorrelation in home-range estimation



- GPS-tracked black bear

(Noonan, Tucker, Fleming, et al., Ecological Monographs. 2019)

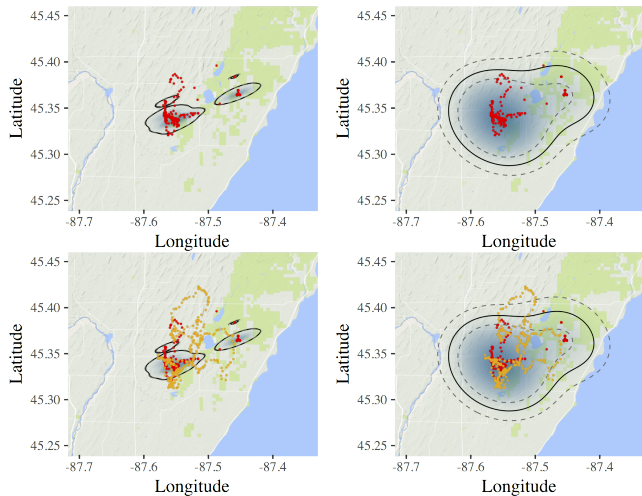
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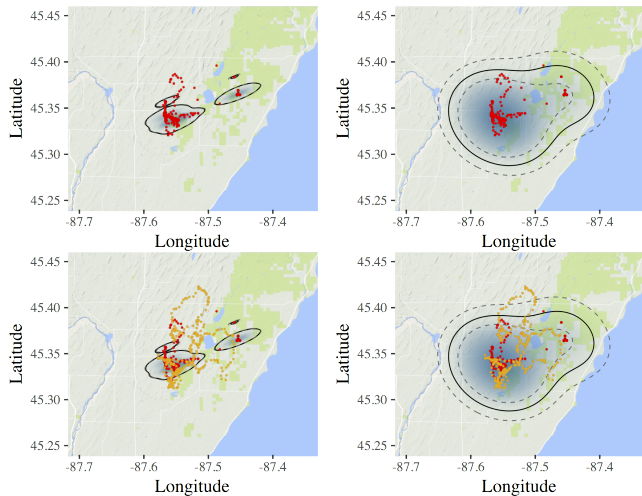
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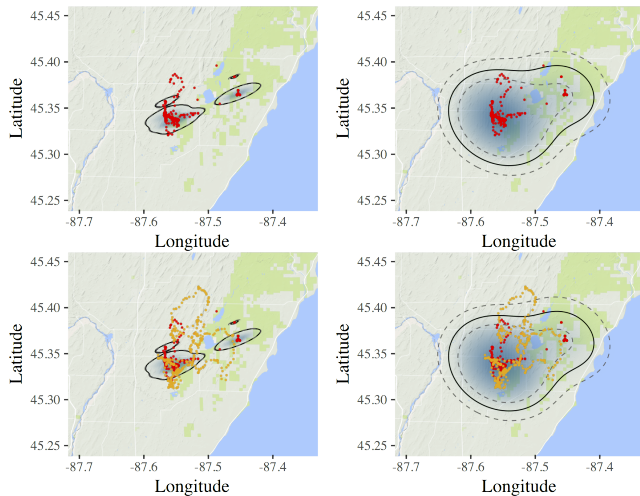
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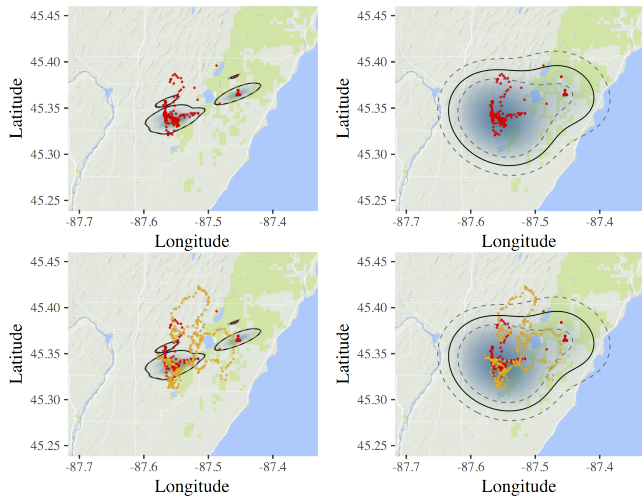
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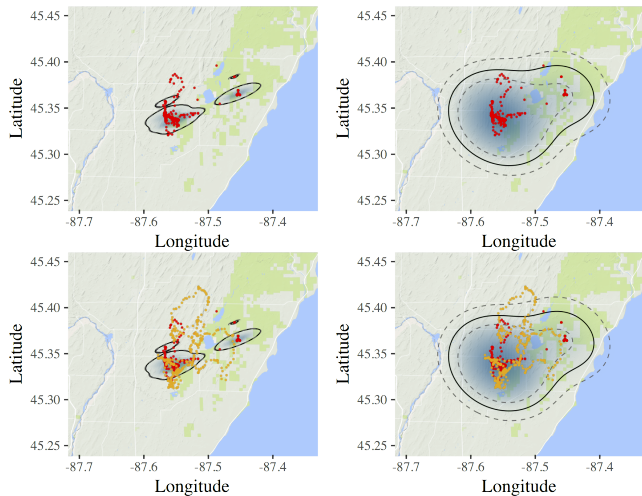
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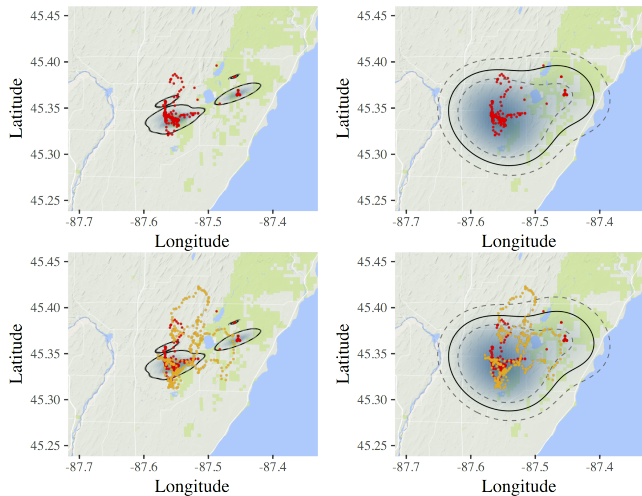
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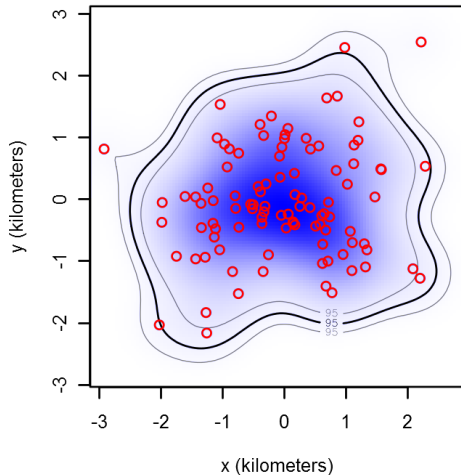


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- Q1: Why does this happen?
- Q2: Does this happen in practice?
- Q3: How did we fix this?

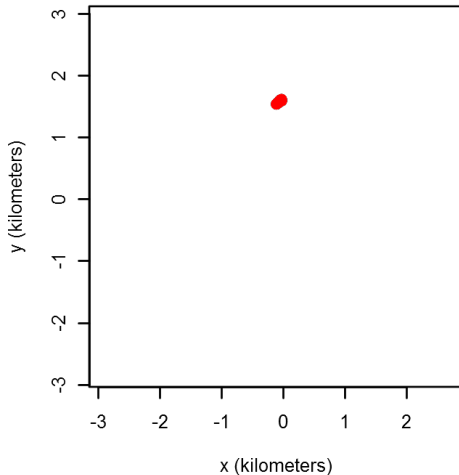
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Q1: Why does this happen?

1 year of data (n=100)



1 hour of data (n=100)



Q2: Does this happen in practice?

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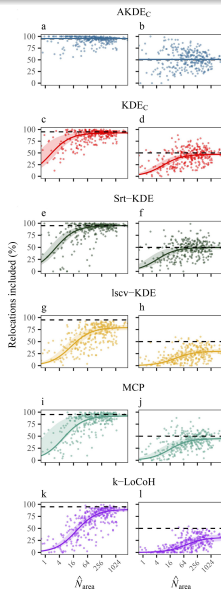
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- Largest comparative analysis to date: 369 individuals, 27 species (Noonan, Tucker, Fleming, et al., Ecological Monographs 2019)

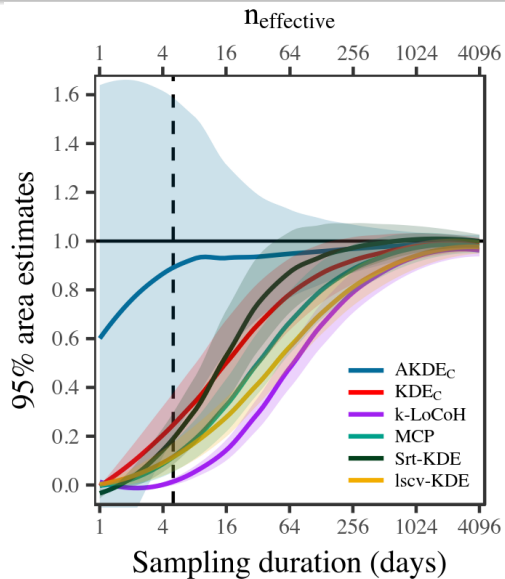
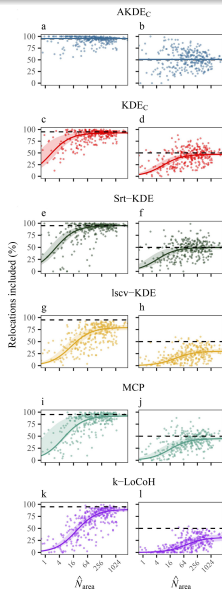
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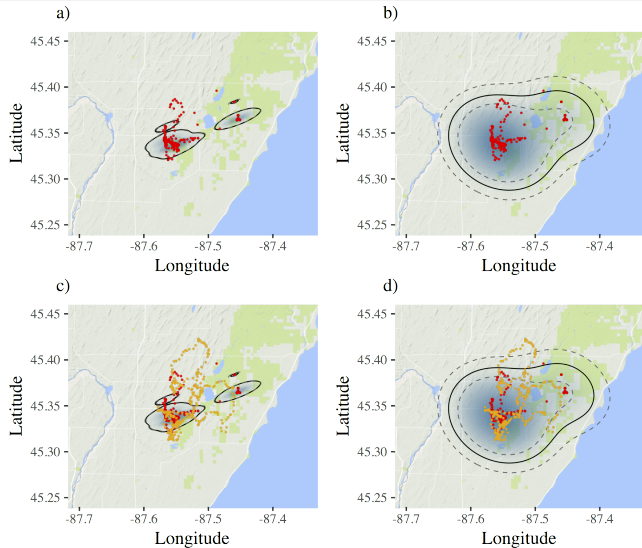
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- Conventional estimates are 2–20 times too small, on average
- Bias is worse for larger species (Noonan, Fleming, et al., Conservation Biology 2020)

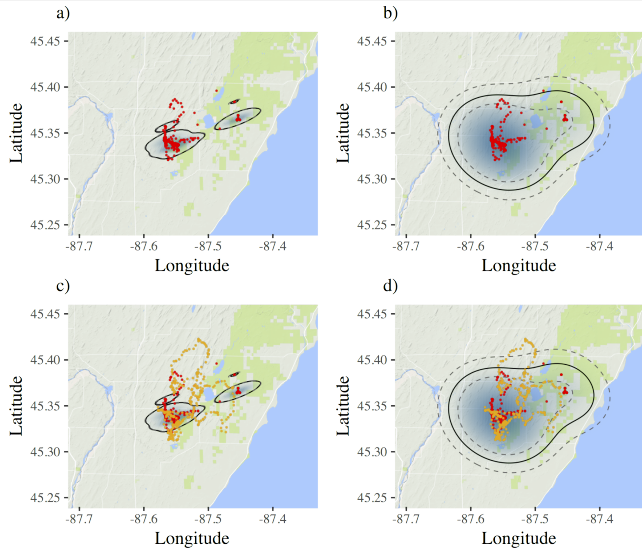
The underestimation of animal space use, the solution



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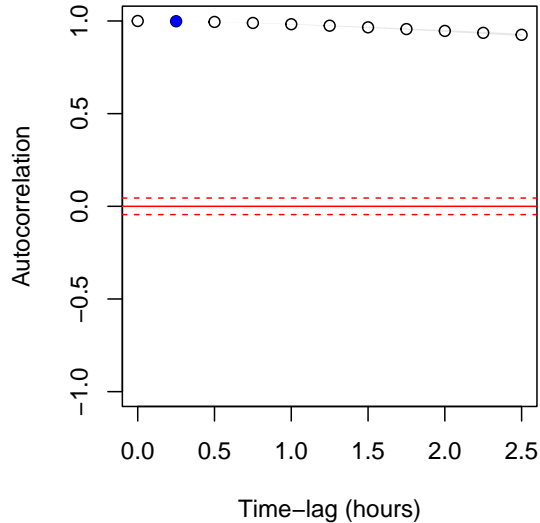
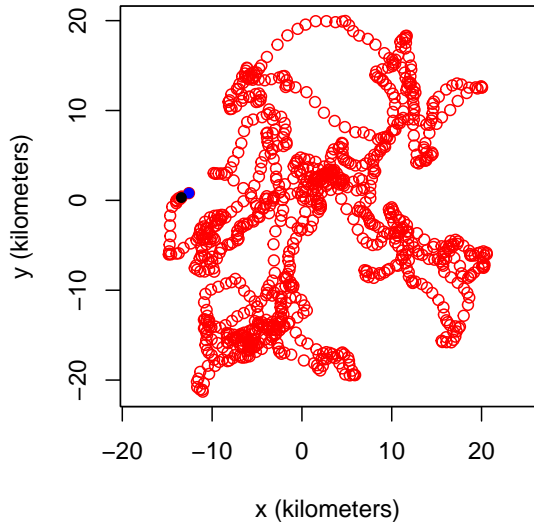
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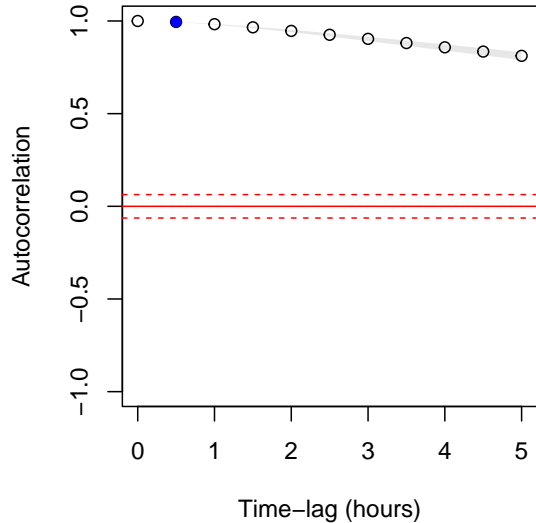
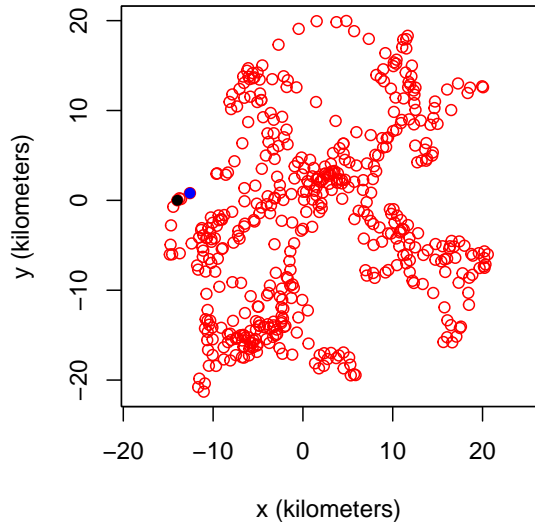
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- A3: Continuous-time stochastic process models of the *autocorrelation*

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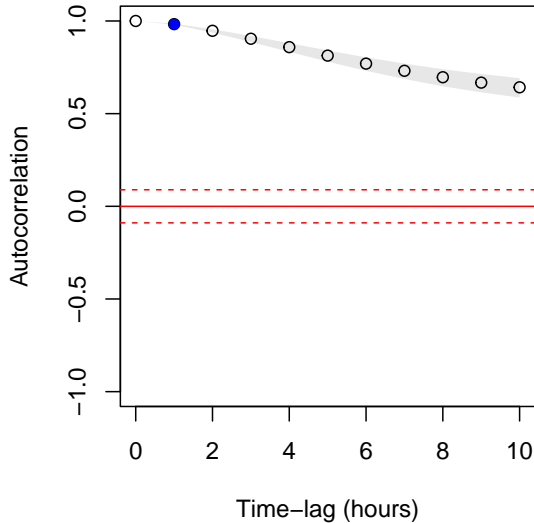
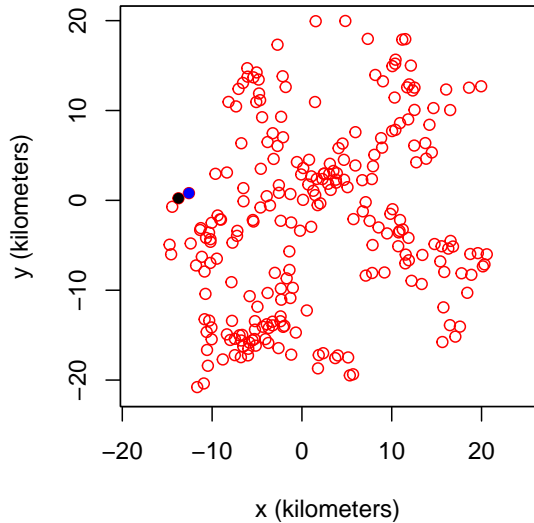
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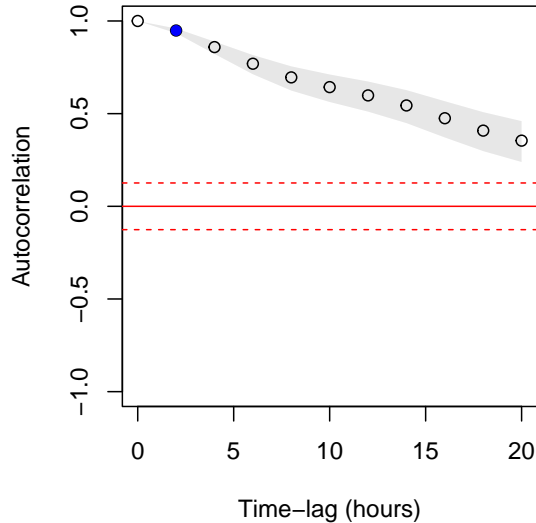
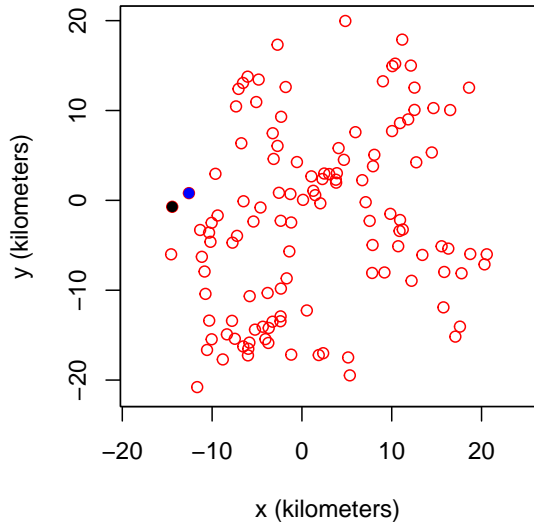
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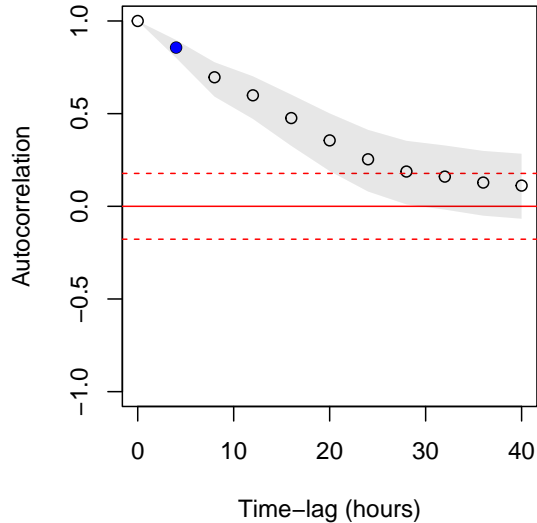
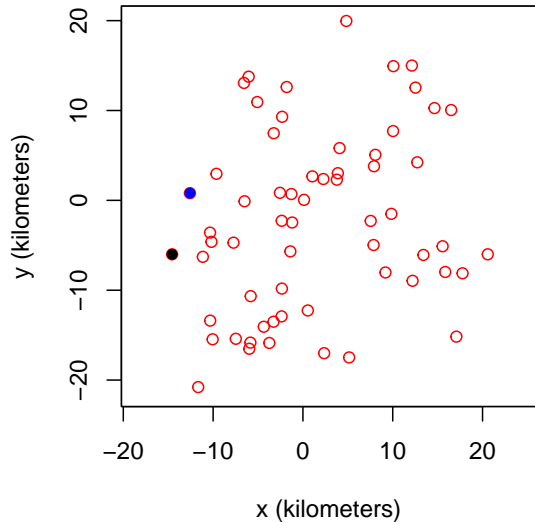
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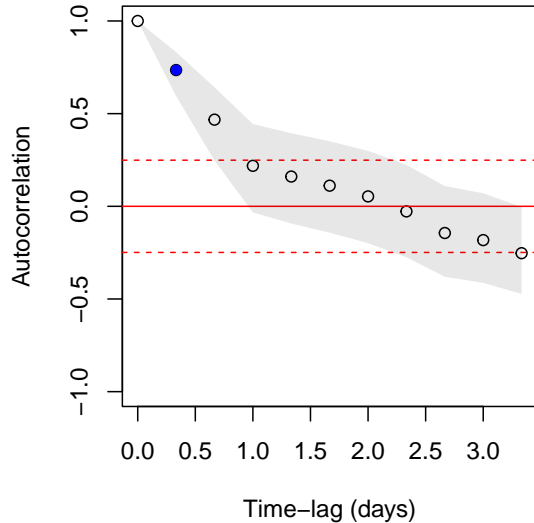
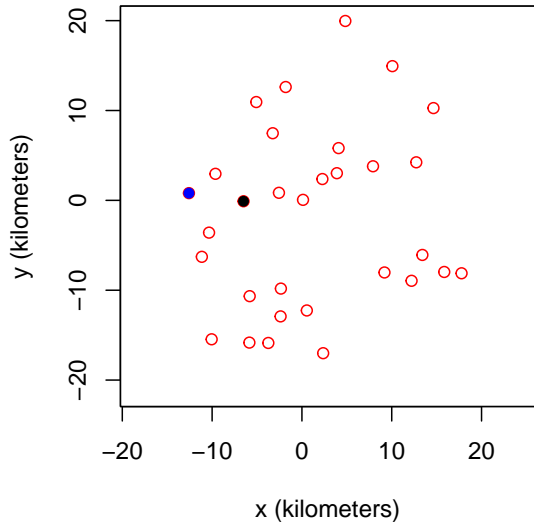
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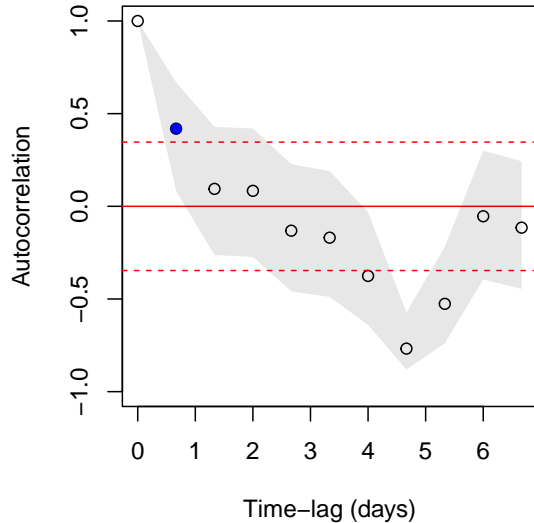
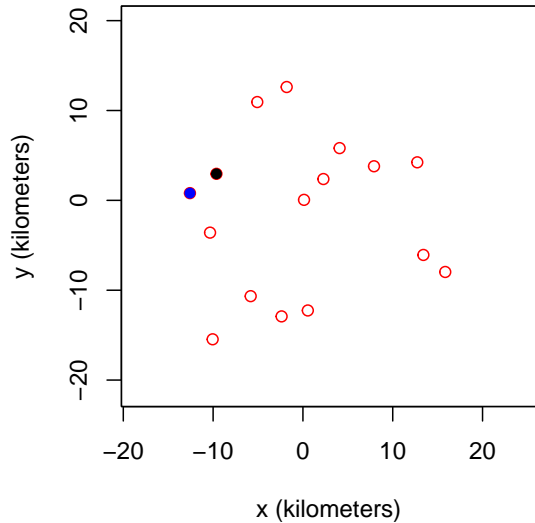
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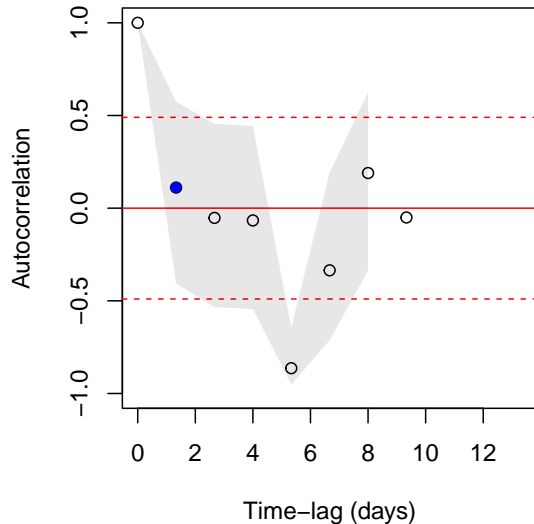
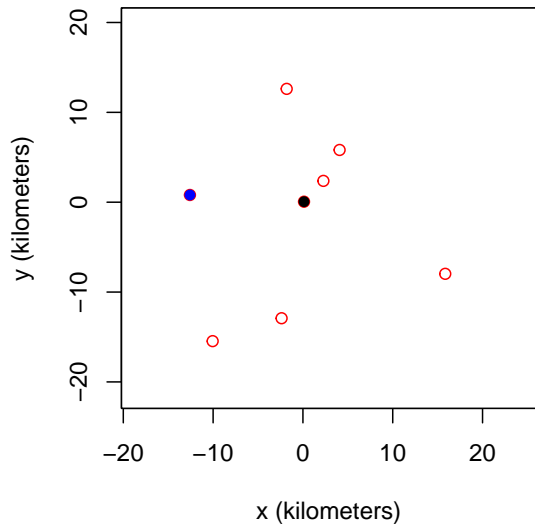
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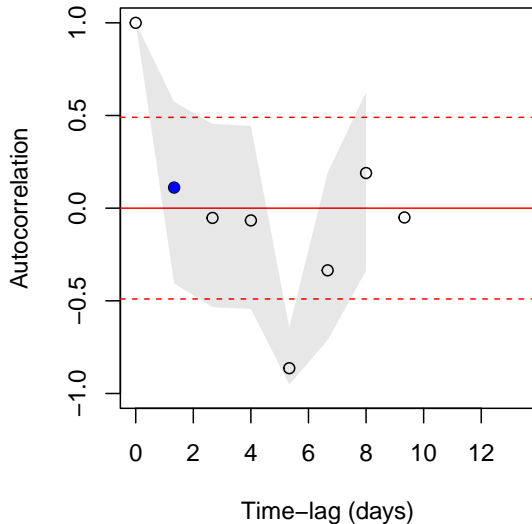
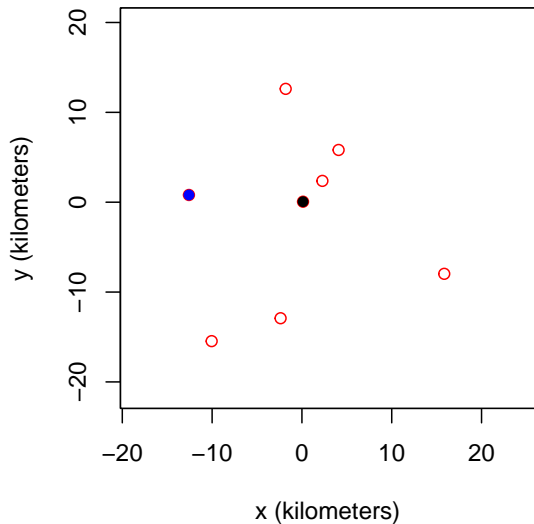
If autocorrelation is the culprit...



...you might have to thin a lot to reach IID

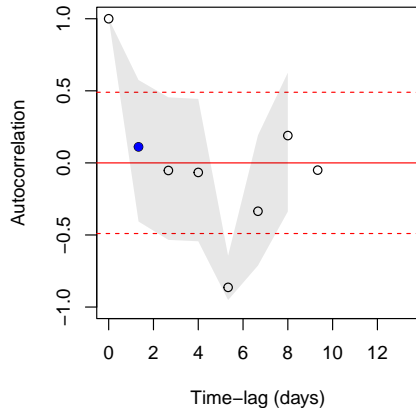
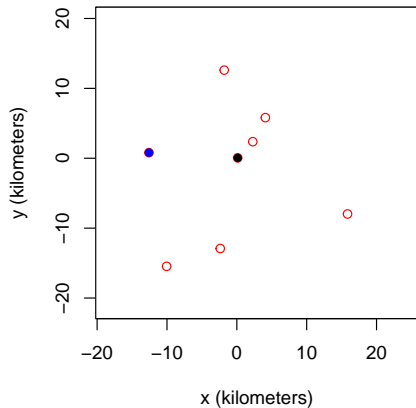


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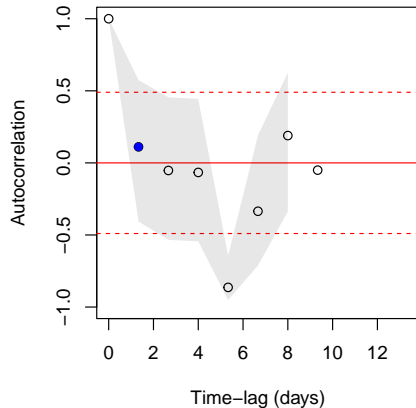
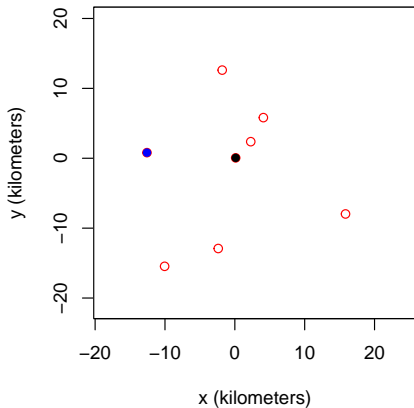
What about speed estimation with these data?

Autocorrelation is information



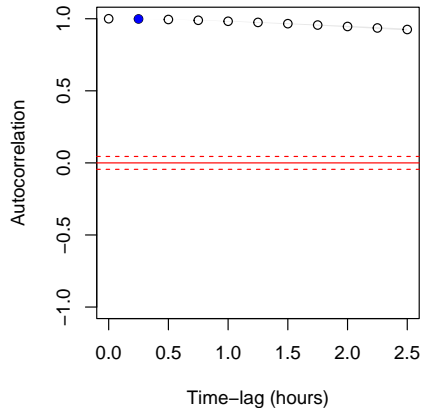
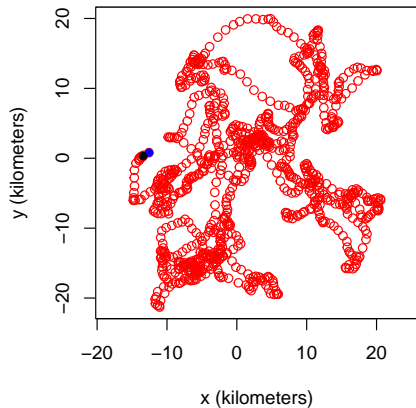
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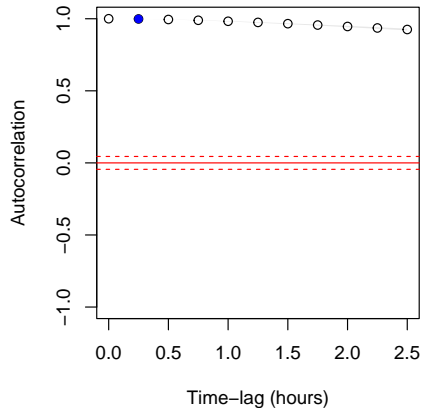
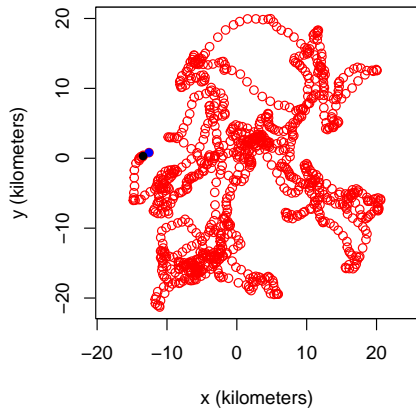
- Convenient for home-range analysis
- Worthless for speed/distance estimation

Autocorrelation is information



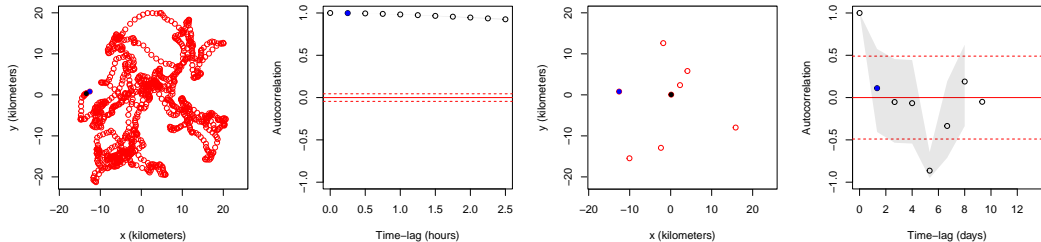
- Inconvenient for (conventional) home-range analysis

Autocorrelation is information



- Inconvenient for (conventional) home-range analysis
- Great for speed/distance estimation

Objective



We want methods that can handle whatever autocorrelation is present in the data

Why continuous time?

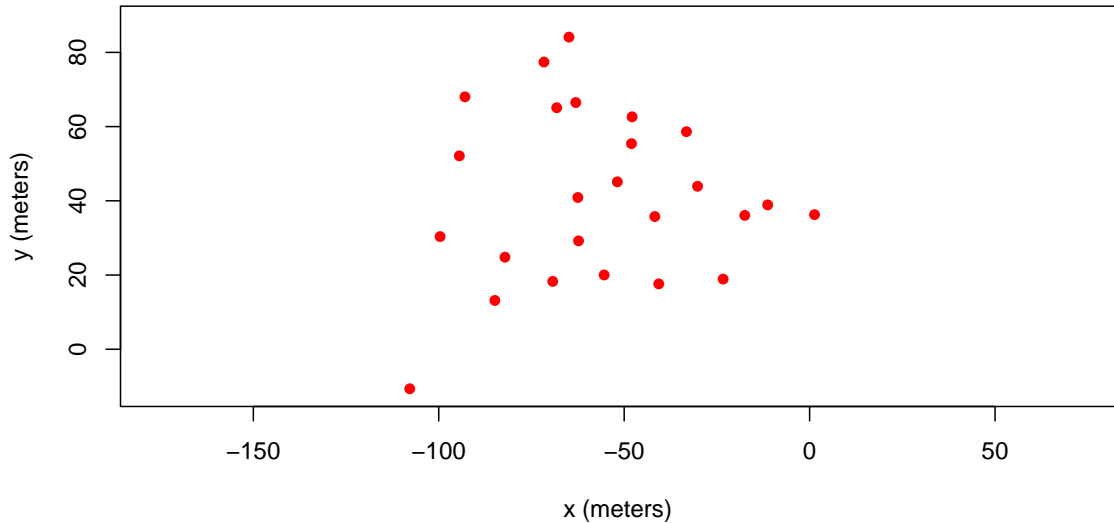
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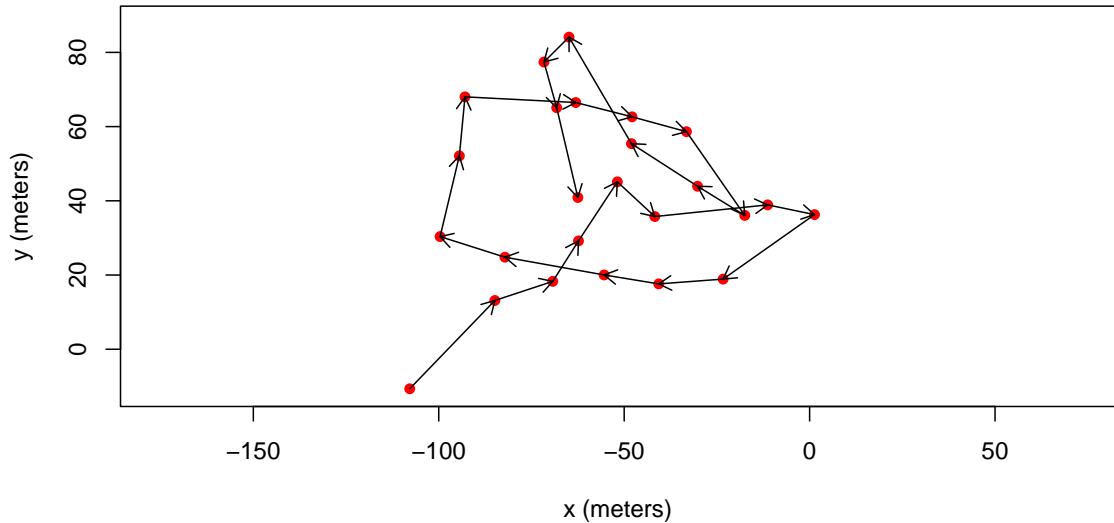
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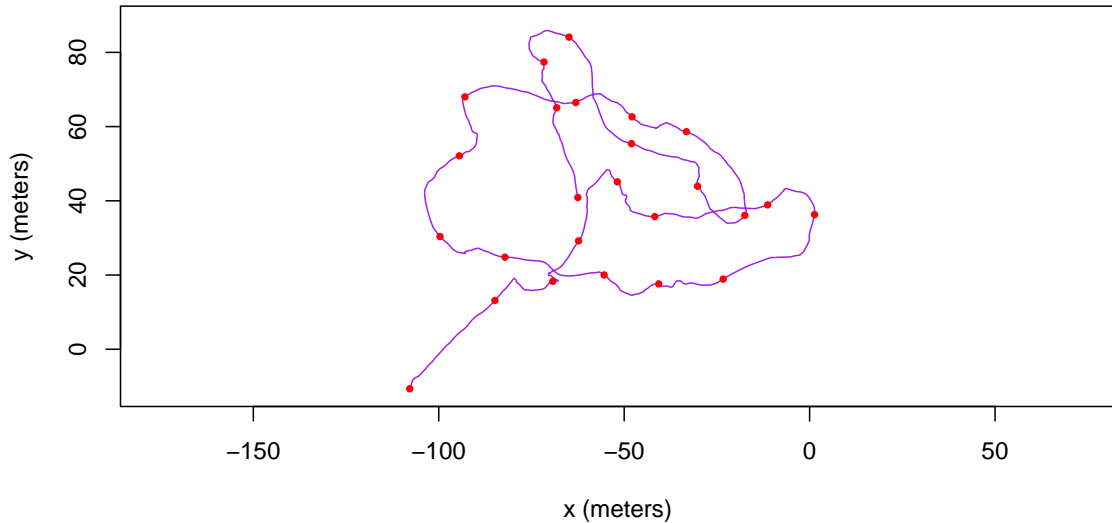
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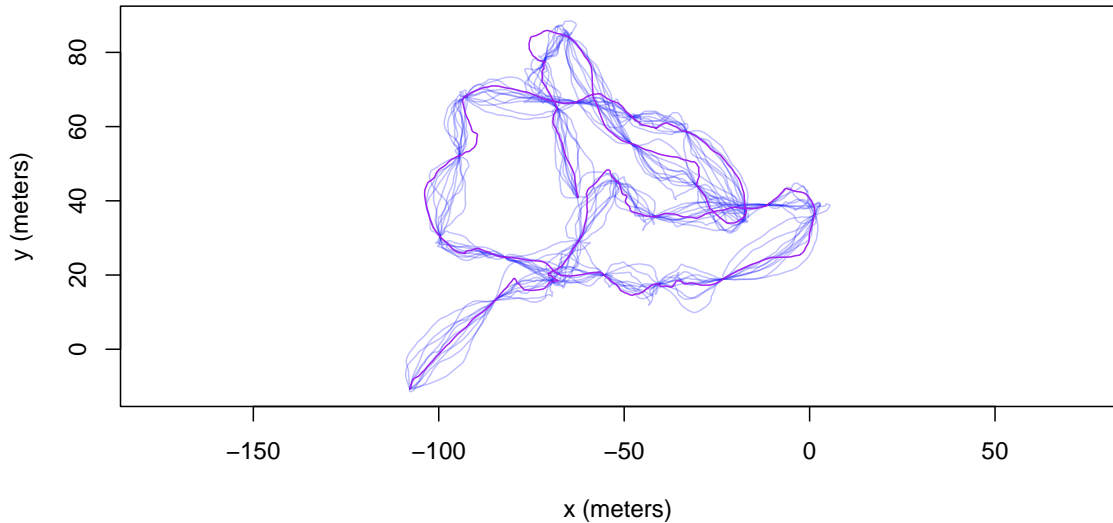
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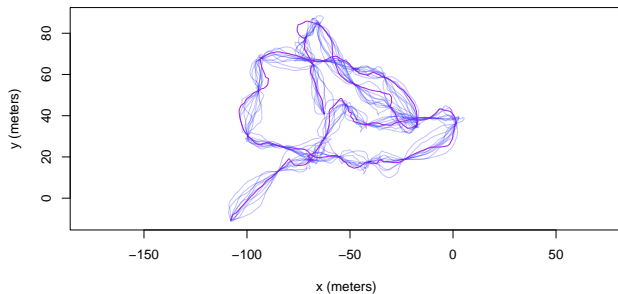
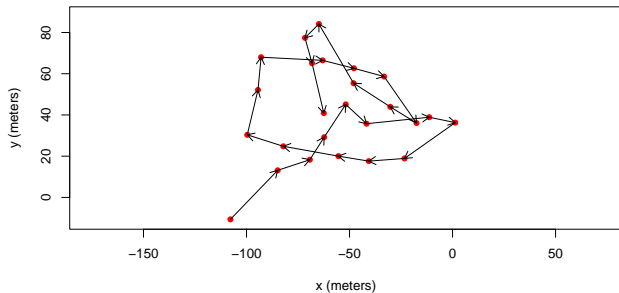
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 - Can accommodate speed, distance, acceleration, power and energy
 - Location error is easy to model (versus CRWs)

Motivating example: Neglecting autocorrelation in speed estimation



Building-block continuous-time stochastic process models

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- Independent locations

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)

Building-block continuous-time stochastic process models

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- Integrated Ornstein-Uhlenbeck motion

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- Integrated Ornstein-Uhlenbeck motion (crawl)
- OUF motion

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- OUF motion

Building-block continuous-time stochastic process models

- Independent locations ($<1\%$)
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion
- OUF motion

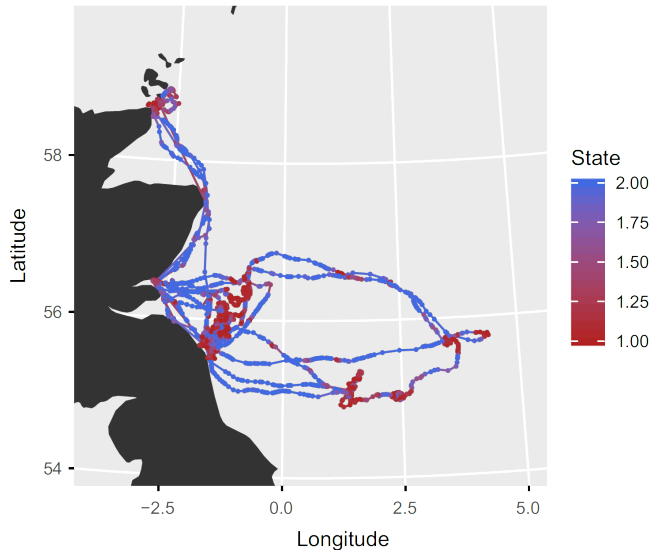
Building-block continuous-time stochastic process models

- Independent locations ($<1\%$)
- Brownian motion
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Coming-soon: Continuous-time behavioral switching models



Michelot & Blackwell (2019)

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Don't assume a model, select a model

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Let's dive into R `ctmmlearn` “`ctmm_intro.R`”